A ray tracer for ophthalmological applications

Abstract  Ray tracing with a personal computer allows realistic simulation of optical properties of the human eye. Patterns of point sources are used as objects. The path of light rays is calculated between the point source and the retina for a Gullstrand eye model with improved parameters; the normal eye model has a resolution limit close to the natural resolution limit of the human eye. The image formed on the retina is projected back to a screen at the distance of the object so as to simulate image interpretation by the brain. Refractive errors are modeled by a change in eye parameters and corrected by eyeglasses or/and contact lenses or by an artificial intraocular lens. For optic correction the parameters of seeing aids can be fitted automatically by a least-squares routine. The effect of faulty eye correction on image quality is visualized by using a photograph of a realistic scene as an object.

Key words  Ray tracing - Mathematical eye model - Tilted intraocular lenses - Shifted contact lenses - Optics


Schlüsselwörter  Strahlendurch- rechnung - Mathematisches Augenmodell - Verkippte Intraokularlinsen Verschobene Kontaktlinsen - Optik
Introduction

The human eye is, among many other things, an optical instrument. Ophthalmologists should know the physics behind this instrument. However, what is, precisely, the physics behind the human eye? In first approximation the eye is a pinhole camera, in second approximation it is a camera with a thin lens and an aperture stop, in third approximation it has four cardinal points, in fourth approximation the photosensitive screen is nonplanar, and a few approximations higher one would have to solve second-order differential equations called eikonal equations. The latter task is impossible to achieve because the refracting field of a real eye is not given in terms of mathematical functions. In physics courses for medical students, teaching does not go beyond cardinal points, which means that it does not even go to a nonplanar screen. Clearly, there is an information gap!

Fortunately, there are computers, and modern personal computers really merit the term computers. What mathematics cannot achieve, namely, to illustrate physics, a computer can do by simulation. A student of physics or medicine has no problem imagining a single light ray: a straight line in a uniform medium, a change in direction at a refractive surface. Finding the precise formula for this change in direction was the great achievement of the Dutch mathematician Willebrord Snell van Roijen (better known as Snellius) 380 years ago. Since then, Snellius' law of refraction has become common sense. Every child who has pointed his flashlight at a swimming pool by night has developed a feeling for refraction. If we use a computer and a mathematical eye model we can calculate hundreds of light rays and show them on a monitor. We see bundles, we can imagine the (caustic) surface, and we see the waist of the bundles. We can check the influence of the aperture stop and the effect of focusing or defocusing on the diameter and position of the waist of the bundles. We get a vivid illustration of how the human eye forms an image and how refractive errors can impair the capacity of image formation.

The first purpose of the present paper is teaching the optics of the human eye by computer simulation. Our computer programs are deliberately kept simple. We use the eye model developed by Gullstrand [6]. We modify it a little but we stick to spherical surfaces. Very simple subroutines describe the straight-line sections of a light ray, find the intersection with an optical surface, and apply the law of Snellius. Then the calling program jumps back to calculate the next straight-line section, and the process is repeated until the light ray intersects the retina. All programs that contain physics are transparent to the reader. The graphics software, the menu, and the optimization routine are less transparent and may be considered black boxes, because one does not have to understand them to understand optical imaging. With nonspherical refractive surfaces, all programs would be black boxes.

The second goal of the present paper is to go beyond teaching and start with practical applications. We allow for deviations from the Gullstrand eye, e.g., myopia and hyperopia; allow for eyeglasses and/or contact lenses; and allow for artificial intraocular lenses. All lenses can be shifted away from the optical axes and/or tilted. At the present stage, however, and for the reason given above, we do not yet allow nonspherical surfaces.

We introduce another crude approximation. Whenever we deviate from a normal eye we have to ask what the patient will see. Nobody sees the image on their retina. They see an image at the position of the object. During babyhood and childhood the brain develops an image interpretation "software" that correlates images on the retina with the outside world. Our present approximation is that this software has been developed with a normal Gullstrand eye and that it has never been modified.

Having mentioned more crude approximations, we must also mention two ingredients that are not trivial. First, we have modified Gullstrand's eye just as much as needed to make the ray-optical resolution limit at the fovea equal to the wave-optical resolution for photopic seeing. Second, we introduce a library routine for parameter fitting. Any one of the parameters that define the optical system, such as dioptries of contact lenses or eyeglasses, can be marked as fitting parameters and any kind of image quality, such as good focusing at the fovea or good focusing at larger angles or a certain image size, can be defined as a fitting goal. The library routine will then vary the optical parameters until it finds a parameter set for which the sum of squared differences between the actual image and the goal becomes minimal.

Another purpose of the present paper is to help patients understand their problem. We want to show them what they will see after optical correction. This means that the computer takes a realistic scene, or a photograph, as an object and creates the images seen by an eye with various seeing aids.

Materials and methods

The eye model

In 1909 Gullstrand [6] introduced his famous eye model, which he called the schematic eye. It has six refractive surfaces: two for the cornea and four for a two-shell crystalline lens (see Fig. 1). All refractive surfaces as well as the retina have spherical shapes, the centers of the spheres being aligned along the optical axis. Gullstrand presented two sets of parameters: one set for an eye focused at the near point and one set for an eye focused at infinity. In Tables 1 and 2 we list Gullstrand's parameters. The spherical surfaces are defined by their radii together with the position of intersection with the optical axis; the front surface of the cornea defines a zero point, positions inside the eye are positive, and negative radii indicate concave surfaces. The notation is exemplified in Fig. 1.

Gullstrand's parameters are remarkably good, but not good enough for our purpose. To allow focusing and to improve resolu-
Table 1  The parameter set of the exact schematic eye presented by Gullstrand, focused at the near point

<table>
<thead>
<tr>
<th></th>
<th>Position [mm]</th>
<th>Radius [mm]</th>
<th>Refractive index</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cornea</td>
<td>0</td>
<td>7.7</td>
<td>1.376</td>
</tr>
<tr>
<td></td>
<td>0.5</td>
<td>6.8</td>
<td>1.336</td>
</tr>
<tr>
<td>Eye lens</td>
<td>3.2</td>
<td>5.33</td>
<td>1.385</td>
</tr>
<tr>
<td></td>
<td>3.8725</td>
<td>2.655</td>
<td>1.406</td>
</tr>
<tr>
<td></td>
<td>6.5725</td>
<td>-2.655</td>
<td>1.385</td>
</tr>
<tr>
<td></td>
<td>7.2</td>
<td>-5.33</td>
<td>1.336</td>
</tr>
<tr>
<td>Retina</td>
<td>24.0</td>
<td>-11.5</td>
<td>-</td>
</tr>
</tbody>
</table>

Table 2  The parameter set of the exact schematic eye presented by Gullstrand, focused at infinity

<table>
<thead>
<tr>
<th></th>
<th>Position [mm]</th>
<th>Radius [mm]</th>
<th>Refractive index</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cornea</td>
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<tr>
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<tr>
<td>Eye lens</td>
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<td>10.0</td>
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<tr>
<td></td>
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<td>7.911</td>
<td>1.406</td>
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<tr>
<td></td>
<td>7.2</td>
<td>-6.0</td>
<td>1.336</td>
</tr>
<tr>
<td>Retina</td>
<td>24.0</td>
<td>-11.5</td>
<td>-</td>
</tr>
</tbody>
</table>

Fig. 1  The eye model presented by Gullstrand [6]; the parameters $S_3$ and $S_7$ for positions, $R_7$ for radius, and $n_2$ and $n_4$ for indices of refraction exemplify the notation

Fig. 2  Rays calculated by the ray tracer; only a small fraction of the calculated rays is shown

Fig. 3  Image formation and image interpretation; the image on the retina is only an intermediate piece of information

The present eye model allows simulation of normal vision in a realistic way and also allows simulation of anametropias. Myopia and hyperopia are created in the model by shifting the position of the retina. We can modify the crystalline lens or replace it, for instance, by an implanted lens of given size and refractive power. We allow for eyeglasses of various shapes and power at various positions. We allow for contact lenses with or without gap filled with tear film. We relax Gullstrand's condition of having all centers of spheres aligned along the optical axis. The seeing aids can be shifted away from the optical axis and can even be tilted. We do not allow, at this stage, nonspherical shapes. This also excludes cylindrical shapes.

Ray tracing

Ray tracing means following the path of a light ray. In our model, the path of light is a straight line until it intersects a refractive surface. At the point of intersection, refraction takes place. The path change
Fig. 4a, b Object and image calculated by the ray tracer. The object is a pattern of point sources (marked by crosses). The image consists of spots formed by rays intersecting the screen.

- Myopic eye corrected by eyeglass of -5.0 D.
- Same eye corrected by contact lens of -4.5 D.

Procedure is repeated until a bundle of many light rays emerges from the given point of the object. The light rays of the bundle are chosen at random by a Monte Carlo routine. If a ray misses the aperture stop the computer discards it because it will not reach the retina. When the number of rays in a bundle reaches a given limit (usually several hundred) the computer will go to the next point of the object. For
For a normal Gullstrand eye with an aperture stop of 2 mm the diameter of the spot is approximately equal to the realistic resolution limit of the human eye. When the dot becomes larger the reason may be either a refractive error or a vision aid that is not ideal. The distance between spots and crossovers can tell us something about distortion effects such as the prism effect of thick eyeglasses.

To get a vivid impression of what a patient with ametropia really sees, we use a realistic scene, or a photograph of a realistic scene as an object. To determine the visual acuity we use Landolt rings of different sizes as objects.

Automatic focusing and optimization

We use a least-squares routine for focusing and for other kinds of optimization. We shall first consider focusing. An image spot, for instance a spot at the fovea, should become as small as possible. First, we calculate its center by the same formula that would be used to calculate its center of gravity. Next, we calculate the sum of squared distances between all the points that form the spot and the center of the spot. Then, we mark one or more parameters of the optical system such as, for instance, the power of an eyeglass or the parameter i mentioned in the eye model. The marked parameter(s) will be varied by the least-squares routine until the sum of squared distance becomes minimal. The least-squares routine is a library routine [9, 10]. The routine implements a steepest-descent method with the special features, e.g., for saving computing time and avoiding traps [7, 11, 12]. For focusing we might want to make a compromise between the fovea, a 10° angle, and a 30° angle. In this case we aim at the sum of squares of a spot at the fovea, a spot at 10°, and a spot at 30°, giving a higher weight to the spot at the center than to the peripheral ones.

Optimization by the least-squares routine is very flexible. We press the difference between an actual image and a desired image, a sum of squares, mark variation parameters, and call the least-squares routine. The routine will give us parameters that bring the image as close as possible to the desired one. An example is discussed below.

Results

The myopic eye

As a first application we investigate a myopic eye. We consider the case of axial myopia by enlarging the distance between the cornea and the retina in our eye model. The defect is corrected by eyeglasses or by contact lenses.

Figure 4 shows an example. In both cases the parameters are those of the normal eye except for the distance between the cornea and the retina, which has been enlarged from 24 to 25.9 mm. For correction with an eyeglass we need -5 D and for that with a contact lens, -4.5 D. These values have been determined automatically by a least-squares routine. The size of the spots shows the optical resolution. One of the dots is blown up 25 times to show how it is composed of 500 intersections of light with the retina. There is considerably less aniseikonia in the case of the contact lens. This feature is shown for various values of the eye length in Fig. 5.

The flexibility of the least-squares routine has been tested by offering an eyeglass together with a contact lens. The routine had to do the focusing and the compensation.
mm the disk resolution reason may al. The dioptria realistic sec. tay of rings of

Fig. 6 Correction by eyeglass (a) and by contact lens (b) of a missing crystalline lens. The eyeglass has +11.75 D and the contact lens has +13.25 D. The combination of an eyeglass of -7.25 D with a contact lens of +9.75 D would completely eliminate the aniseikonia

Correction of aphakia

Figure 6 shows the effect of eyeglass correction and contact-lens correction of aphakia. The eyeglass has +11.7 D and the contact lens has +13.25 D. A combination of an
Fig. 7 Image of a photograph seen and interpreted by the following eyes: a the normal Gullstrand eye, b a myopic eye with a -4.5-D contact lens displaced by 20°, c a myopic eye with a -5.0-D eyeglass tilted by 20°, and d an artificial intraocular lens tilted by 20°. The angle of 20° is unrealistic in all cases shown but clearly exhibits the distortion.

eyeglass with -7.25 D and a contact lens with +19.75 D would eliminate the aniseikonia completely.

Decentered contact lenses, prism effect of eyeglasses, and dislocated and tilted artificial intraocular lenses

Figure 7 exhibits the effects of various eye modifications on images of a realistic scene. The object is a photograph. The image is composed of pixels like the ones on a color television screen. Details of Fig. 7 are given in the legend.

We should mention a technical detail. When the object is composed of point sources and the screen is continuous we proceed as described above in ray tracing. When the object is continuous and the screen has pixels, as in the present case, we let the computer calculate the path of light rays in the reverse direction. We start from a pixel, go to the retina (which is a continuous screen in our model), and proceed from there to the object. The computer then transfers the information bright/dark and color from the object to the pixel. Also, to economize on computer time we sacrifice some accuracy and take only around 40 rays/pixel instead of several hundred. Of course, the path of a light ray is the same when calculated in the reverse direction. Therefore, the imaging process may still be called ray tracing.

Discussion

By taking a linear superposition of two sets of parameters given by Gullstrand we obtained a model eye that has a resolution limit comparable with that of the normal human eye.
under photopic conditions. Ametropias (refractive errors) can be simulated by modifying model parameters. Wearing eyeglasses and/or contact lenses may be added artificially. Artificial lens can be inserted. All lenses can be shifted from the optical axis and/or tilted.

A computer code that calculates images by ray tracing many rays has been presented and several examples been given for illustration. A simple model for interpretation by the human brain has also been presented.

The main purpose of the present work is to aid teaching computer simulation. Bundles of rays exhibit the process of retinal image formation in an illustrative way. The present computer code may also be considered a first step toward application. Image qualities can be defined as a sum of squared deviations between the actual image and a desired ideal image. A least-squares fit of optical parameters will then be performed automatically.

The code consists of well-documented FORTRAN subroutines called by a TURBO PASCAL main program. A menu allows easy access. The least-squares fitting routine is taken from Moré et al. [12]. The code is available free of charge for educational, private, and research purposes only. (Interested readers should send a short message to wolfgang.fink@uni-tuebingen.de to get the address of the ftp-server, which provides the code.)

References